Theory and Practice of Uniform Interpolation

Sam van Gool[1]

IRIF, Université Paris Cité, Paris, France

A formula I is called an *interpolant* of an entailment $A \vdash B$ if $A \vdash I$, $I \vdash B$, and I only uses propositional variables that appear in both A and B. The property that any entailment admits an interpolant is known as the (deductive) *interpolation property*, and holds for many logics, including classical and intuitionistic propositional logic.

An interpolant is called *right uniform* if it can be chosen independently from the formula on the right of the entailment. More precisely, I is a *right uniform interpolant* for a formula A with respect to a propositional variable p if I is p-free and the formula I is an interpolant for every p-free formula B such that $A \vdash B$. The concept of a *left uniform interpolant* is defined analogously. By an argument similar to the one for 'usual' interpolants, one obtains left and right uniform interpolants for any formula in classical propositional logic. In the case of *intuitionistic* propositional logic, a surprising theorem of A. Pitts [3] says that left and right uniform interpolants still exist, although they are more complicated to compute.

The aim of this talk is to survey a number of recent results related to uniform interpolation in propositional logics, involving its algebraic, topological, model-theoretic, and computational aspects. More specifically, I hope to explain: how uniform interpolation can be studied via adjunctions between congruence lattices [4]; how Pitts' theorem can be proved via an open mapping theorem for Esakia spaces [5]; how recent work on mechanization allows to compute uniform interpolants for various intuitionistic and modal logics [1, 2]. Throughout the talk, I will also point to some open problems that this work has led to.

References

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